QUESTION BANK :

Different type of questions (Mathematics Honours Core Course wise) are given below

Mathematics (Honours)
Paper-CC1T

1. Find the oblique asymptotes of the curve $y=\frac{3x}{2}\log(e - \frac{1}{3x})$.
2. If $y=e^{ax}\cos^2 bx$ then find y_n (a>0,b>0)
3. If $y = x^{n-1}\log x$,then proved that $y_n = \frac{(n-1)!}{x}$.
4. What is reciprocal spiral? Sketch it.
5. Find the centre and foci of the conic

   $X^2 -2y^2 -2x +8y -1 =0$

6. Find the equation of the sphere of which the circle xy+yz+zx=0,x+y+z=3 is a great circle.

7. Find the condition that the line $\frac{1}{r} = A\cos\theta + B\sin\theta$ may touch the conic $\frac{1}{r} = 1 - e\cos\theta$.

8. For what angle must the axes be turned to remove the term xy from $7x^2 +4xy +3y^2$.

9. Find the integrating factor of the differential equation

   $(2xy +3x^2y+6y^2)dx +(x^2+6y^2)dy=0$.

10. Show that the general solution of the equation $\frac{dy}{dx} +Py =Q$ can be written in the form y=k(u-v)+v, where k is a constant and u and v are two particular solution .

11. Solve $\frac{dy}{dx} +y\cos x =xy^n$.

12. Solve $(px^2+y^2)(px+y) = (p+1)^2$ [u=xy ,v=x+y]

13. Reduce the equation $X^2 +3y^2 +3z^2 -2xy -2yz -2zx+1=0$ to its canonical form and determine the type of the quadratic represented by it .

14. Find the reduction formula for $\int \cos^m x \sin(nx)\, dx$.

15.If $I_n = \int_0^{\frac{\pi}{2}} \cos^{n-2} x \sin x \, dx$, n>2 Prove that $2(n-1)I_n = 1+(n-2)I_{n-1}$.

16. When the axes are turned through an angle the expression (ax + by) becomes (a′x′ + b′y′) referred to new axes ; Show that $a^2+b^2= a'^2+b'^2$.

17. Show that the semi-latus rectum of a conic is a harmonic mean between the segments of the cylinder whose generators are parallel to the straight line $\frac{x}{-1} = \frac{y}{2} = \frac{z}{3}$ and whose guiding curve is $x^2+y^2=9$, z=1.

18. Determine the order and degree of the differential equation

i) $\{1+ (dy/dx)^2\}^{1/2} = 1+ x$

ii) $(d^2y/dx^2)^2 + y = dy/dx$

19. Find the equation of the sphere described on the join of P(2,-3,4) and Q (-5,6,-7) as diameter .

20. Find the differential equation of all circles , which pass through the origin and whose center are on x-axis.

21. Solve and test for singularity of the equation $p^3 - 4xyp + 8 = 0$.

22. Find the nth order derivative of $e^{(ax+b)}\sin x$.

23. Trace the curve $y=1/x^3+3$.

24. Find $\lim_{x\to 0}$ (tanx-x)/(x-sinx).

25. Find the values of c for which the plane x+y+z=c touch the sphere $x^2+y^2+z^2 -2x -2y-2z-6 = 0$.

26. If $x^{2/3}+y^{2/3}=c^{2/3}$ is the envelope of the family of curves $x^2/a^2+y^2/b^2=1$, then prove that the parameters a and b are connected by the relation a+b=c.

27. Show that the straight line $r\cos(\theta-\alpha)$ =p touches the conic $l/r = 1+e\cos\theta$ if $(l\cos\alpha - ep)^2 + l^2\sin^2\alpha = p^2$.

28. Find the integrating factor of the differential equation $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$ and solve it.

29. Find the equation to the cone whose slope at any point is equal to y+2x and which passes through the origin.

30. Define exact differential equation. Examine whether

(cosy +ycosx)dx + (sinx –xsiny)dy = 0 is an exact differential equation.

Mathematics (Honours)
Paper-CC2T

1. Show that the equation
   $(x-a)^3 + (x-b)^3 + (x-c)^3 + (x-d)^3 = 0$, where a,b,c,d are not all equal, has only one real root.
2. Let $S = \{(x,y,z) \in R^3 : x+y+z=0\}$ .prove that S is a subspace of R^3 .
3. Define an eigen vector of a matrix $A_{n \times n}$ over a field F .Show that there exist many eigen vectors of corresponding to an eigen value $\lambda \in F$.
4. Let V be a real vector space with $\{\alpha, \beta, \gamma\}$ as a basis . Prove that the set $\{\alpha+\beta+\gamma, \beta+\gamma, \gamma\}$ is also a basis.
5. Show that the intersection of two subspaces of a vector space over a field F is a subspace of V.
6. Find the condition on $a,b \in R$ so that the set $\{(a,b,1),(b,1,a),(1,a,b)\}$ is linearly dependent in R^3.
7. For what value of k the planes x-4y+5z=k, x-y+2z=3 and 2x+y+z=0 intersect in a line ?
8. Find the remainder when 777^{777} is divided by 16.
9. If k be a positive integer ,show that gcd (ka,kb) =k gcd (a,b)
10. Prove that for all $n \in N$, $(2+\sqrt{3})^n + (2-\sqrt{3})^n$ is an even integer.
11. State and prove Cayley - Hamilton theorem .
12. Find a basis for the vector space R^3 that contains the vectors (1,2,0) and (1,3,1).
13. Find the basis and dimension of the subspace W of R^3 where W= $=\{(x,y,z) \in R^3 : x+2y+z=0$ $2x+y+3z=0\}$.
14. A linear mapping $T: R^3 \to R^3$ is defined by T(x,y,z) = (2x+y-z,y+4z,x-y+3z), $(x,y,z) \in R^3$. Find the matrix of T relative to the ordered bases {(1,0,0),(0,1,0),(0,0,1)} of R^3.
15. Solve the system of equation y+z=a, x+z=b , x+y = c and use the solution to find the inverse of the matrix $A=\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.
16. Determine The Condition for which the system of Equations

    x+y+z=b

    2x+y+3z=b+1

    $5x+2y+az=b^2$

    a) has only one solution, b) has no solution, c) has many solutions.
17. Find the all real λ for which the rank of matrix A is 2 :

$$A=\begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 3 & \lambda \\ 1 & 1 & 6 & \lambda+1 \end{bmatrix}$$

18. Prove that there is a one to one correspondence between the sets (0,1) and [0,1].
19. Prove that the product of any n consecutive integers is divisible by n.
20. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ given that the roots are in G.P.
21. If a,b,c be the roots of the equation $x^3 + px^2 + qx + r = 0$,then form the equation whose roots are a+1/a ,b+1/b ,c+1/c .

22. Solve the equation by Cardan's method $x^3 - 15x^2 - 33x + 847 = 0$
23. Solve the equation by Ferrari's method $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$.
24. If f: A $\longrightarrow$ B and g: B $\longrightarrow$ C be two mappings such that $g \circ f : A \longrightarrow C$ is surjective then show that g is surjective.
25. For any vector space V , Prove that Rank (T) +Nullity (T)=Dimension (V).
26. Let V and W be two vector spaces over a field F. Let T: V $\longrightarrow$ W be a linear mapping. Then T is injective if and only if Kernel T $=\{\theta\}$.
27. Z is a complex number satisfying the condition $\left|z - \frac{3}{z}\right| = 2$. Find the greatest and least value of $|z|$
28. State and proved De-Moivre's theorem.
29. Use the theory of congruences to proved that $7 \mid 2^{5n+3} + 5^{2n+3}$ for all $n \geq 1$.
30. Use Division algorithm to prove that the square of any integer is of the form 5k or $5k \pm 1$, k is an integer.

Mathematics (Honours)
Paper-CC3T

1. Define uncountable set with example.
2. State the Archimedean property.
3. Define isolated points of a set with example .
4. Define bounded sequence .
5. State the Bolzano-weierstrass theorem for a set.
6. Define interior points of a set. Find the interior points of the set $S = \{x \in R : 1<x<3\}$
7. Prove that the number $\sqrt{2} + \sqrt{3}$ is irrational.
8. Show that the finite collection of open sets is open.
9. Prove that union of an enumerable number of enumerable sets is enumerable.
10. If $S= \{ (-1)^m + \frac{1}{n} : m,n \in N\}$, then find the limit points of the set S.
11. Define derive set of a sets S with example.
12. If a sequence $\{a_n\}$ converges to l , then prove that the sequence $\{|a_n|\}$ converges to $|l|$.
13. If $\{u_n\}$ be a bounded sequence and $\lim_{n\to \alpha} v_n =0$ then $\lim_{n\to\infty} u_n v_n =0$.
14. Define closed set .if S be a non-empty bounded subset of R ,then prove that Sup S and Inf S belong to S . Find Sup S and Inf S of the set .
15. State Bolzano – Weierstrass theorem for sequence . Given an example of a bounded sequence .
16. Using D'Alembert's ratio test to examine the convergence of the series $\sum_{n=1}^{\infty} \frac{4^n}{4n}$
17. If $\{x_n\}$ convergence to l then prove that the sequence $\{(x_1 + x_2 +....+ x_n)/ n \}$ converge.
18. Prove that a monotonic increasing sequence , which is bounded above is convergent and converges to its exact upper bound .
19. State and prove Cauchy's General principle of convergence theorem.
20. Examine the convergence of the series $\sum u_n$, where $u_n= \frac{(n+1)(n+4)}{n(n+2)(n+5)}$.
21. Every convergent sequence is bounded.
22. Define Cauchy's sequence. Examine the sequence { 1/n} is a Cauchy sequence.
23. Every non-empty set of real number that is bounded below has an infimum in R.
24. Show that if $S=\{\frac{1}{n} : n\in N\}$ then Inf S =0
25. If $x_n = \frac{1}{n} \sin\frac{n\pi}{2}$, show that the sequence $\{x_n\}$ convergence.

Mathematics (Honours)

Paper-CC4T

1. Let $W(y_1, y_2)$ be the wrongkian of two linearly independent solutions y_1 and y_2 of the equation

$\frac{d^2y}{dx^2}$ + p(x) $\frac{dy}{dx}$ +q(x) y = 0 then prove that $W(y_1, y_2)$ p(x) = $y_2\frac{d^2 y_1}{dx^2}$ - $y_1\frac{d^2 y_2}{dx^2}$.

2. Solve the differential equation $\frac{d^2y}{dx^2}$ + a^2y = sec (ax) by the method of variation of parameters.
3. State and prove that super position principle for homogeneous linear differential equation .
4. Solve :$\frac{dx}{3y-2z} = \frac{dy}{z-3x} = \frac{dz}{2x-y}$.
5. Find the equilibrium point of the system of differential equations . $\dot{x}=e^{x-1}$ -1 and $\dot{y}$ $=ye^x$.
6. Solve the differential equation
   $\frac{d^2 y}{dx^2} - 3\frac{dy}{dx} = x + e^x \sin x$ by the method of undetermined coefficient.
7. Show that (yz+ xyz) dx +(xz+xyz) dy + (yx + xyz)dz =0.
8. Find the power series solution of the equation
   $(x^2 -1)\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + xy = 0$ given that y(0) =4 and $\frac{dy}{dx}$ =6 at x=0.
9. Solve :
   $\frac{dx}{x^2(y-z)} = \frac{dy}{y^2(z-x)} = \frac{dz}{z^2(x-y)}$
10. Solve :
    $(y^2 + yz)\, dx + (z^2 + zx)dy + (y^2 - xy)\, dz = 0$.
11. Find the value of k where the vectors $2\hat{\imath}-\hat{\jmath}+\hat{k}$, $\hat{\imath} +2\hat{\jmath}-\hat{k}$ and $k\hat{\imath}-4\hat{\jmath} +5\hat{k}$ are coplanar .
12. Find the condition of coplanar and non- coplanar of three vectors.
13. A necessary and sufficient condition that a proper vector $\vec{u}$ has a constant length is that $\vec{u}.\frac{d\vec{u}}{dx} = 0$.
14. Prove that , if $\vec{a}$, $\vec{b}$,$\vec{c}$ are coplanar then $[\vec{a}\vec{b}\vec{c}] = 0$.
15. Four points whose position vectors are $\vec{a}$,$\vec{b}$, $\vec{c}$, $\vec{d}$ are coplanar iff
    $[\vec{a}\vec{b}\vec{c}] = [\vec{b}\vec{c}\vec{d}] + [\vec{c}\vec{a}\vec{d}] + [\vec{a}\vec{b}\vec{d}]$
16. If $\vec{\omega}$ be a constant vector , $\vec{r}$ and $\vec{s}$ are vector functions of a scalar variable t and if $\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$, $\frac{d\vec{s}}{dt} = \vec{\omega} \times \vec{s}$, then show that
    $\frac{d}{dt}(\vec{r} \times \vec{s}) = \vec{\omega} \times(\vec{r} \times \vec{s})$.
17. Reduce the expression **(b + c). {(c+a) ×(a + b)}** in its simplest from and prove that it vanishes when **a ,b ,c** are coplanar.
18. Find the derivatives (with respect to t) of the following :
    $r^2\mathbf{r} + (a.\mathbf{r})b$ ii) $\frac{\mathbf{r}\times\mathbf{a}}{\mathbf{r}.\mathbf{a}}$

19. Show that $\frac{d}{dt}$ ($\mathbf{r}\times \frac{d\mathbf{r}}{dt}$) = $\mathbf{r}\times \frac{d^2\mathbf{r}}{dt^2}$

20. Evaluate $\int_0^1 (\boldsymbol{r} \times \frac{d^2\boldsymbol{r}}{dt^2})dt$ where $\mathbf{r} = t^3\hat{\imath} + 2t^2\hat{\jmath} + 3t\hat{k}$.

21. Find the phase curve of the dynamical system of equations $\dot{x}$ = 3x –y and $\dot{y}$ = -4y .also describe the nature of stationary point.

22. Define ordinary point , singular point and regular singular point of a second order linear differential equation.

23. Solve : $\frac{dx}{dt} - 7x + y = 0$, $\frac{dy}{dt} - 2x - 5y = 0$

24. Find the general solution of the differential equation

    $(D^2 + 1)y = x^2\cos x$.

25. Find the particular integral of the differential equation $(D^2 + 1)\, y = \sin x$.

Mathematics (Honours)
Paper-CC5T

1. Define discrete metric space.
2. Is a singleton set open in a discrete metric space? Give reasons.
3. State Rolle's theorem on differentiability.

4. Let f:[0,1] $\to$[0,1] be a continuous function on [0,1] ,prove that there exist a point c in [0,1] such that f(c) =c.

5. State Taylor's theorem with Cauchy's from of remender.

6.A function f is differentiable on [0,2] and f(0)=0,f(1)=2,f(2)= 1.Prove that f'(c)=0 for some c in (0,2).

7. State Cauchy's Mean Value Theorem on differentiability .

8. State and prove the Maclaurin's theorem with Cauchy's from of remainder.

9. Let I be a bounded interval and f:I $\to$R be a uniformly continuous on I . Then prove that f is bounded on I . Show that $\cos\frac{1}{x}$ is not uniformly continuous.

10. Let f:R $\to$R be strictly increasing and continuous and let S=f(R) .Then f^{-1}:S$\to$R is also strictly increasing and continuous .

11. Let

$$f(x) = 1, x \in Q$$
$$= 0, x \in Q^c.$$

Show that f is continous on Q.

12. Let f(x) = $\begin{Bmatrix} 2, x \in Q \\ 0, x \in R-Q \end{Bmatrix}$,prove that f is discontinuous at every point c in R.

13. Prove that $\frac{\tan x}{x} > \frac{x}{\sin x}$, $0<x<\frac{\pi}{2}$.

14. Verify Rolle's theorem for $f(x) = x^2$, $x \in [-1,1]$.

15. Use Mean Value Theorem to prove that $|\sin x - \sin y| \le |x - y|$ for all x,y $\in$ R.

16. A function f is twice differentiable on [a,b] and f(a) = f(b) =0 and f(c) <0 for some c in (a,b) . Prove that there is at least one point ξ in (a,b) for which $f''(\xi) > 0$.

17. Is the function, $f(x) = x^2$, x $\in$ [0; 2], uniformly continuous on [0, 2] ?

18. State Rolle's theorem on differentiability .

19. Find the local extremum points of the function

$$f(x) = x^2/(1-x)^3 .$$

20. Let f:R $\to$R is defined by f(x) = $|x| + |x-1|$,x$\in$ R . Find derived function f' and specify the domain of f'.

21. Give an example of functions f and g which are not continuous at a point $c \in R$ but the sum f+ g is continuous at c.

22. Let $D \subset R$ and f and g be functions on D to R and $g(x) \neq 0$ for all $x \in D'$. Let $c \in D'$ and $\lim_{x \to c} f(x) = l$, $\lim_{x \to c} g(x) = m \neq 0$. Then prove that , $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{l}{m}$.

23. Use MVT to prove the inequality

$\sin^{-1} x < x$ if $0 \leq x < 1$ .when does the equality hold .

24. What do you mean by removable discontinuity of a function at an interior point of an interval ?

25. State Cauchy's criteria for the existence of $\lim_{x \to \alpha} f(x)$.

Mathematics (Honours)
Paper-CC6T

1. Define cyclic group with example.

2. Let G be a group and $a \in G$, $o(a)=12$. Find $o(a^3)$ and $o(a^8)$.

3. Let $(G,\circ)$ be a group and $a,b \in G$. If $a^2 = e$ and $a \circ b^2 \circ a = b^3$, prove that $b^5 = e$.

4. Define center of a group G . Find center of S_3 .

5. Show that the groups (Q, +) and (R,+) are not isomorphic .

6. State and prove first isomorphism theorem of groups.

7. Prove that every group of order p^2 is abelian , where p is prime.

8. Find the number of element of order 5 in $Z_{15} \times Z_5$.

9. Let $(G,\circ)$ be an abelian group and n be a fixed positive integer. Let $H=\{a^n : a \in G\}$. Prove that $(H,\circ)$ is a subgroup of $(G,\circ)$.

10. Prove that the order of a permutation on a finite set is the l.c.m of length of its disjoints cycles .

11. Let $G=S_3$, $G'=(\{1,-1\},.)$ and $\phi : G \to G'$ is defined by

$\phi(\alpha) = 1$ if α be an even permutation in S_3

$= -1$ if α be an odd permutation in S_3.

If ϕ is homomorphism, determine Kerϕ.

12. Let G be a group and $a \in G$. Define a mapping $\phi : Z \to G$ by

$\phi(n) = a^n$, $n \in Z$. Show that ϕ is homomorphism

13. If $\phi : G \to G'$ be a group homomorphism , prove that $\phi(e)=e'$ and $\phi(x^{-1}) = \phi(x)^{-1}$. $\forall x \in G$.

14. Let G be a group and the mapping $\phi : G \to G$ is defined by $\phi(x) = x^{-1}$

,$x \in G$. Prove that ϕ is an automorphism if G is abelian.

15. Find the number of generators of a cyclic group of order 72.

16. Find the all isomorphisms from the group $(Z_8, +)$ to $(Z_6 ,+)$.

17. Prove that in a finite group G, order of any subgroup divides order of the group G . Does the converse true ? Justify your answer with example .

18. Prove that if H has index 2 in G , then H is normal in G.

19. Prove that every group of prime order is cyclic.

20. Show that alternating group of symmetric group of degree three is normal subgroup.

21. Write down all the elements of the factor group G/H and also Cayley table :

G= Z_6 and H= {0, 3} .

22. let G be a cyclic group of order 12 generated by a and H be the cyclic subgroup of G generated by a^4 . Prove that H is normal in G . Verify that the quotient group $\frac{G}{H}$ is a cyclic group of order 4.

23. Prove that the alternating group A_3 is a normal subgroup of S_3 .

24. Is multiplication of permutations commutative? Justify with example.

25. Find the all subgroups of the group Z_{10} .

Mathematics (Honours)
Paper-CC7T

1. Show that Δ (f(x)g(x)) = f(x) Δ g(x) + g(x+h) Δ f(x)
2. Show that the Lagrangian function are invariant under a linear transformation .
3. Proved that $H_r^{(n)} = H_{n-r}^{(n)}$
4. Define degree of precision of a numerical integration formulae . write the degree of precision of trapezoidal rule and simpson's one third rule.
5. Establish Lagrange's interpolation formula. Show that the Lagrangian functions are invariant under a linear transformation.
6. Show that Newton – Raphson method is said to have a quadratic rate of convergence .
7. Write down the sufficient condition of gauss-seidel iteration method .
8. State fourth order runge-kutta method .
9. Write down the geometrical interpretation of simpson's one third rule.
10. Derive Trapezoidal rule from general quadrature formula and discuss its geometrical significance.
11. . What do you mean by relative error and percentage error?
12. Why relative error is a better indicator of the accuracy of a computation then the absolute error? Explain with an example.
13. Prove that $\Delta\nabla$ f(x) = Δ f(x) - ∇ f(x) where Δ and ∇ carry their usual meaning.
14. Given that f(0)=2, f(1)=4, f(2)=6, f(3)=10 and 3^{rd} difference being constant. Find f(5).
15. Prove that the sum of Lagrangian's function or coefficient is unity.
16. Show that divided differences are symmetric function of their arguments.
17. Write down the principle of numerical differentiation and numerical Integration.
18. Write down the composite from of Trapezoidal rule.
19. Define 'Degree of Precision' of a numerical integration formulae.
20. Discuss the condition of convergence of Newton-Raphson method.
21. Prove that a divided difference is symmetric function of its arguments. If $f(x)=x^2$ then prove that $f[x_0, x_1, x_2]$ is constant and all higher order difference are zero.
22. State the difference between direct and iterative methods.
23. Describe Gauss Jacobi method for solution of a system of linear equation. State the sufficient condition for convergence of this method.
24. Solve the following equations by Gauss-Jordan elimation method:

    $x_1 + x_2 + x_3 = 3$

    $2x_1 + 3x_2 + x_3 = 6$

    $x_1 - x_2 - x_3 = -3$
25. Derive the expression for Secant method to find the root of an equation.

Mathematics (Honours)

Paper-CC8T

1. State and prove Darboux theorem.

2. If a function f: [a,b]→ R be integrable on [a,b] then prove that | f | is integrable on [a,b]. Is the converse true?

3. State Dirichlet test for the convergence of an improper integral.

4. State Weierstrass M-test for the uniform convergence of a series of function.

5. State Dirichlet's conditions for convergence of a Fourier series.

6. Let f(x) be the sum of a power series $\sum a_r x^r$ on (-R,R) where R>0. If f(x)+f(-x)=0 for all x∈ (-R,R). Prove that a_r=0 for all even positive integer.

7. Let $\sum a_n x^n$ be a power series with radius of convergence R(>0). Let f(x) be sum of the series on (-R,R) then prove that f(x) is continuous on (-R,R).

8. Obtain Fourier series representation of f in [-π,π] where f(x)=x ∀ x∈[-π,π] and hence deduce that $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\ldots\ldots=\pi/4$.

9. A function f is defined on [1,3] by f(x)=[x²]. Evaluate $\int_1^3 f(x)\,dx$.

10. If a function f: [a,b]→R be R integrable on [a,b] and f(x)≥0 for all x∈[a,b],then prove that $\int_a^b f \geq 0$.

11. Define Riemann sum for a function f. A function f is defined on [0,1] by

    f(x) = 1, if x is rational
    = 0, if x is irrational

    Using Riemann sums show that f is not integrable on [0,1].

12. Prove that Γ(n+1)= n Γ(n), n>0.

13. Examine the convergence of the integral $\int_0^1 \log x/\sqrt{(1-x)}\,dx$.

14. State Bonnet's form of second mean value theorem of integral calculus. Hence establish $\left| \int_a^b \sin x^2\,dx \right| \leq 1/a$ in 0<a<b<∞.

15. If $f_n(x)=x^n$,x∈[0,1], show that the sequence of functions {fn} is not uniformly convergent on [0,1].

16. State and prove Cauchy criterion for the uniform convergence of sequence of functions.

17. Let D ⊂R and for each n∈N, fn: D→R is continuous function on D. If the series ∑fn be uniformly convergent on D then prove that the sum function S is continuous on D.

18. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \mathrm{anx}^{\mathrm{n}}$ where an= 2^n+3^n, n≥1;

19. Find the sum of the power series $1+x+x^2+\cdots\cdots\cdots$ on its interval of convergence. Deduce the power series expansion log(1-x) and use Abel's theorem to prove that 1-1/2+1/3-1/4+············= log2.

20. State and prove fundamental theorem of integral calculus.

21. Let f be defined on [0,3] by f(x)=[x],x∈[0,3]. Show that f is integrable on[0,3] but $\int_0^3 f$ can not be evaluated by the fundamental theorem.

22. $Is \sum_{n=1}^{\infty} (\mathrm{sinnx}/\sqrt{\mathrm{n}})$ is a Fourier series or not? Justify.

23. Using comparison test, show that $\int_0^1 (x^{p-1}/1+x)$ is convergent if p>0 and is divergent if p≤0.

24. Define Logarithmic function. Use it to prove that (i) for x>0,y>0,L(xy)=L(x)+L(y) and (ii) 2<e<3.

25. Let f: [a,b]→R, g: [a,b]→ R be both integrable on [a,b]. Prove that f+g is integrable on [a,b] and $\int_a^b (f+g)= \int_a^b f+ \int_a^b g$.

Mathematics (Honours)
Paper-CC9T

1. State and prove Euler's theorem on homogeneous function in three variables .
2. Is f(x,y)=|y|(1+x) differentiate at (0,0)?
3. Find the maximum or minimum value of

   $f(x,y)=x^3 +y^3 - 3axy$.
4. Show that the function $f(x,y) = (x-y)^3 + (2-x)^2$ has a saddle point at (2,2).
5. Define Gradient, Divergence and Curl of a vector point function .
6. Find the equation of the tangent plane to the surface xyz = 4 at the point (1,2,2).
7. State and prove Green's theorem in a plane .
8. State and prove sufficient condition for differentiability of a function f(x,y) at a point (a,b).
9. State and prove the schwarz's theorem.
10. Let f(x,y) =1/2, y=rational

    =x, y= irrational

    Verify whether $\int_0^1 dy \int_0^1 f(x,y)dx$ exists or not.

11. State and prove sufficient condition for differentiability.

12. Find the shortest distance from the origin to the hyperbola $x^2 +8xy +7y^2 =225$, z=0.

13. Find the directional derivative of $\varphi = x^2yz + 4xz^2$ at (1,-2,-1) in the direction of $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.

14. Calculate the normal derivative of the function φ = yz + zx +xy at (1,1,2) and find the unit vector normal to the surface $x^2 - y^2 + z = 2$ at point (1,-1,2).

15. State Gauss's divergence theorem and give its geometrical interpretation.

16. Show that a necessary and sufficient condition for a scalar point function φ to be constant is that grad $\varphi = \vec{0}$.

17. Changing the order of integration , show that

$$\int_0^1 dx \int_x^{\frac{1}{x}} \frac{y\,dy}{(1+xy)^2\,(1+y^2)} = \frac{\pi-1}{4}.$$

18. Define solenoidal field and irrotational field . If **A** anb **B** are irrotational , prove that **A×B** is solenoidal .

19. Find the total work done in moving a particle in a force field given by

$\vec{F} = (2x-y+z)\,\hat{\imath} + (x+y-z)\hat{\jmath} + (3x-2y-5z)\hat{k}$, along a circle C in the xy-plane $x^2 + y^2 = 9$, z=0.

20. Verify the divergence theorem for $\vec{F} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, z=0 and z=3.

21. If u= $\tan^{-1}\frac{x^3+y^3}{x-y}$ then prove that $x\frac{\partial u}{dx} + y\frac{\partial u}{dy} = \sin 2u$.

22. Show that f(x,y) = $\sqrt{|xy|}$ is not differentiable at origin .

23. Evaluate $\iint e^{\frac{x}{y}}$ dx dy over the region D= {(x,y) : $1 \leq y \leq 2 , y \leq x \leq y^3$}

24. Evaluate $\iiint 2x\, dv$ over the region 2x+ 3y+z =6 that lies in the first octant .

25. Show that $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential.

Mathematics (Honours)
Paper-CC10T

1. In a ring (R,+,.), prove that a.(-b)=(-a).b= -(a.b) for all a,b $\in$ R.

2. Prove that the characteristic of an integral domain is either zero or a prime number.

3. Define characteristic of a ring. If in a ring (R,+,.), $a^2=a$ for all a$\in$ R, prove that charR=2 .

4. Prove that every field is an integral domain. Is the converse true? Justify with example.

5. Let R be a ring. The centre of R is the subset Z(R) defined by Z(R)={x$\in$ R: xr=rx for all r$\in$R}. Then prove that Z(R) is a subring of R.

6. Define prime ideal and maximal ideal in the ring R.

7. Prove that the rings Z and 2Z are not isomorphic.

8. Find the field of quotients of the integral domain Z.

9. Define homomorphism of rings. Let R=(Z,+,.) and ϕ:R$\rightarrow$R be defined by ϕ(x)=2x,x$\in$Z. Examine ϕ is homomorphism or not.

10. Prove that in a field F, $a^2 = b^2$ implies either a=b or a= -b for a,b in F.

11. Define basis and dimension of a vector space.

12. Find a basis for the vector space R^3, that contains the vectors (1,2,1) and (3,6,2).

13. Find the dimension of the subspace S of R^3 defined by

S= {(x,y,z)$\in$ R^3: 2x+y-z = 0 }.

14. Examine if the set of vectors {(2,1,1),(1,2,2),(1,1,1)} is linearly independent in R^3.

15. Define nullity and rank of a linear mapping.

16. Prove that the linear mapping T: $R^3 \rightarrow R^3$ defined by T(x,y,z)=(x+y,y+z,z+x), (x,y,z)$\in R^3$ is one to one and onto.

17. Determine the linear mapping T: $R^3 \rightarrow R^3$ which maps the basis vectors (0,1,1),(1,0,1),(1,1,0) of R which maps the basis vectors (0,1,1),(1,0,1),(1,1,0) of R^3 to (1,1,1),(1,1,1),(1,1,1) respectively. Verify that dim KerT+ dim ImT= 3.

18. Let V and W be vector spaces over a field F. Show that a linear mapping T: V→ W is invertible if and only if T is one-to-one and onto.

19. A linear mapping T: $R^3 \to R^3$ is defined by T(x,y,z) = (x-y,x+2y,y+3z), (x,y,z)∈R^3. Show that T is non singular and determine T^{-1} .

20. In R^3, α=(4,3,5), β=(0,1,3), γ=(2,1,1). Examine if α is a linear combination of β and γ .

21. Let R be a ring with unity I, having no divisor of zero. Let a,b ∈ R and ab=I. Prove that ba=I too.

22. Examine if the ring of matrices { $\begin{pmatrix} a & b \\ 2b & a \end{pmatrix}$: a,b∈ R} is a field.

23. Let T: $R^3 \to R^3$ be defined by T(x,y,z)=(x+1,y+1,z+1), (x,y,z)∈R^3. Examine if T is linear.

24. Show that the cancellation law holds in a ring R if and only if R has no divisor of zero.

25. Let { α,β,γ } be a basis of a real vector space V and c be a non zero real number. Prove that (α+cβ, β+cγ, γ+cα} may not be a basis of V.

Mathematics (Honuours)
Paper-CC11T

1. Form the differential equation by eliminating the function f & F,
   $Y = f(x - ct) + F(x+ct)$
2. State the cauchy – kowalewkaya theorem .
3. Derive the relation pv=h, the symbols have their usual meanings.
4. Reduce to canonical form of the given partial differential equation

   $U_x – U_y = U$.

5. Solve the differential equation $3\partial u/\partial x + 2\partial u/\partial y = 0$ given that $u(x.0)=4e^{-x}$.
6. Classify the given partial differential equation
   i)$yr +(x+y)s + xt = 0$

   ii)$\partial^2z/\partial x^2 = x^2\partial^2z/\partial y^2$

7. Solve : $xu_x + yu_y = 0$
8. Write the differential equation of the central orbit in pedal form.
9. A particle describes the parabola $p^2=ar$ under a force, which is always directed towards its focus. Find the law of force.
10. Define linear partial differential equation with example.
11. Solve : $(y^2 + z^2 –x^2)p – 2xyp + 2xz = 0$.
12. What is the degree and order of a given partial differential equation $\partial^2z/\partial x^2 =(1+ \partial z/\partial xy)^2$
13. Find the partial differential equation by eliminating arbitrary constant $Z= (x-a)^2 + (y-b)^2$, where a and b are arbitrary constant.
14. A particle describes an ellipse under a force which is always directed towards the centre of the ellipse. Find the law of force.
15. Define Lagrange's equation with example.
16. Write the Keplar's Law of planetary motion.
17. Classify the following partial differential equation
    $\partial^2u/\partial x^2 + 4\partial^2u/\partial x\partial y + 4\partial^2u/\partial y^2 =0$

18. Solve: $y^2z/x\, p + xzq = y^2$

19. Find the canonical from of a given partial differential equation

    $U_x - u_y = u$

20. Solve: $z(xp-yq) = y^2 – x^2$

21. Define semi linear partial differential equation with example.

22. Reduce the equation $3\partial^2u/\partial x^2 + 10\partial^2u/\partial x\partial y + 3\partial^2u/\partial y^2 = 0$ to its canonical form and hence solve it.

23. Derive the one dimensional wave equation.

24. A machine gun of mass M_0 stands on a horizontal plane and contains a shot of total mass m which is fired horizontally at a uniform rate with constant velocity u relative to the gun.

25. Find the partial differential equation of all spheres of constant radius and having centre on the xy-plane.

Mathematics (Honours)
Paper-CC12T

1. Find the number of elements of order 5 in $Z_{25} \times Z_5$.
2. Is $Z_3 \times Z_5$ isomorphic to Z_{15} ? why?
3. State Sylow's third theorem with example .
4. Define the commutator subgroup of a group. Prove that a commutator subgroup of a group is a characteristic subgroup of the group.
5. If G is a group , prove that Aut(G) is also a group under the operation function composition .
6. Find all subgroups of order 3 in $Z_9 \times Z_3$.
7. State the fundamental theorem of finite abelian groups. Use it to classify all abelian groups of order 540.
8. Let G be a noncyclic group of order 21 . How many sylow's 3-subgroups does G have?
9. Prove that a simple group of order 60 has a subgroup of order 6 and a subgroup of order 10.
10. Find all abelian groups(upto isomorphism) of order 108.
11. Let G be a group acting on a non-empty set S. Then show that this action of G on S induces a homomorphism from G to A(S).
12. Compute Aut(G), Where G={1,-1,i,-i}
13. If $G=S_3$, Then proved that $G'=A_3$, where G' be the commutator subgroup of G.
14. Define Internal and External direct product with correct example.
15. Let G be a finite group and H be a Sylow-p-subgroup of G. Then prove that H isa unique Sylow p-Subgroup if and only if H is normal in G.
16. Show that the direct product $Z \times Z$ is not cyclic.
17. Find all non- isomorphic abelian group of order 35.
18. Define class equation and find the class equation of S_3.
19. State Sylow's 3^{rd} theorem and give a supporting example.
20. Define simple group and show that the group of order 175 is not Simple.
21. Find the conjugacy classes in the dihedral group D_4 and write down the class equation.
22. Deduce the class equation of S_3.
23. Let G be a group of order pq where p,q are primes. Then prove that G can't be simple.
24. Prove that in a finite group G, the number of elements in the conjugacy class of $a(\in G)$ is a divisor of O(G).
25. Find the number of inner automorphisms of the Symmetric group S_3 .

Mathematics (Honuours)
Paper-CC13T

1. Prove that in any discrete metric space, the sets are closed.

2. Show that a convergent sequence in a metric space (X,d) is a Cauchy sequence.

3. Define sequence and uniform continuity in a metric space.

4. Show that Q is a disconnected set in (R,d) where d is the usual Metric on R.

5. Define finite intersection property.

6. Let X=C[a,b] and d(x(t),y(t)) $=_{t\in[a,b]}^{sup}|x(t)-y(t)|$. Show that in (X,d) the sequence $\{x_n\} \rightarrow x$ iff $\{x_n(t)\} \rightarrow x(t)$ uniformly on [a,b].

7. Let X = Q = the set of all rational number and d(x,y) =$|x-y|$ for all x,y $\in Q$. Show that (Q,d) is an incomplete metric space.

8. Let (x,d) be a metric space and A$\subseteq X$ and f : (X,d)$\rightarrow \mathbb{R}$ defined by f(x)=d(x,A) for all x$\in X$.Show that f is uniformly continuous function.

9. State and proved Heine Borel theorem.

10. Let (X,d) be a complete metric space and a function f:(X,d)$\rightarrow (X,d)$ satisfying d(f(x),f(y)) < d(x,y) for all x,y$\in X$ with x$\neq y$.Show that f has a unique fixed point.

11. Show that f(z) = $|z|^2$ is continuous for all z $\epsilon\mathbb{C}$.

12. Show that $\lim_{z\rightarrow -1} \frac{iz+3}{z+1}$ = ∞ .

13. Let X=Q = the set of all rational number and d(x,y) = $|x-y|$, for x,y∈ Q .Show that (Q,d) is an incomplete metric space.

14. Define harmonic function and analytic function.

15. Let $\lim_{z\rightarrow z_0} f(z)$ = l ,Show that $\lim_{z\rightarrow z_0} \overline{f(z)} = \bar{l}$.

16. Show that $\lim_{z\rightarrow \tau\tau\propto} \frac{4z^2}{(z-1)^2}$.

17. State and prove the Hausdorff property .

18. Let f be an analytic function on D ⊆ ℂ with Im(f) =0 .show that f is a constant function on D.

19. State and prove Cauchy- Goursat's theorem .

20. For the point $\frac{1}{\sqrt{3}} - i\frac{1}{\sqrt{3}}$ on $\mathbb{C}$ find the corresponding point on RIEMAN SPHERE S: $x^2 + y^2 + z^2 = 1$, under stereographic projection.

21. Let f(z) = u + iv be an analytic function ,show that the curves u = constant , v = constant represented on z-plane , intersect at right angle .

22. Show that if the limit of f(z) exist , then its unique .

23. Show that f(z) = $\bar{z}$ is not differentiable any where in $\mathbb{C}$.

24. Let (X,d) be a metric space and A, B $\subseteq$ X and A be connected set with A$\subseteq$ B $\subseteq \bar{A}$. Show that B is connected set in (X,d).

25. Let (X, d) be a compact metric space and a function f: (X,d) $\rightarrow$ (X,d) satisfying $d(f(x),f(y)) < d(x,y)$,for x,y $\in$X with x$\neq$ y.Show that f has unique fixed point.

Mathematics (Honuours)
Paper-CC14T

1. Let D be an integral domain and a, b, c ∈ D with a ≠0 , b≠ 0 .Show that a/b ,b/c ⟹ a/c
2. Define multiplicative norm function .
3. Show that Z[$\sqrt{-5}$] is not a unique factorization domain .
4. Define irreducible element with example .
5. Define general form of Eirenstein criterion .
6. Let V be a real inner product space and S_1 ,S_2⊆ V with S_1⊆ S_2 ,show that ${S_2}^{\perp} \subseteq {S_1}^{\perp}$.
7. Let V= R^3 with standard inner product and P= L{(1,1,0) ,(0,1,1) } . find $P^{\perp}$.
8. Find the minimal solution of x + 2y – z =19.
9. Define dual space and dual basis .
10. Let D be an integral domain and a.b ∈ D with associates in D iff a/b , b/a.
11. Show that in a unique factorization domain every irreducible element is a prime element .
12. Show that an Euclidian domain is a principle ideal domain.
13. Let W be a subspace of V. show that V = W⊕ $W^{\perp}$.
14. Find the minimal solution of

    x + 2y + z =4

    x- y + 2z = -11

    x + 5y = 19.
15. Show that Cyclotonic polynomial is irreducible over Z.
16. State and prove Cauchy – Schwarz inequality .
17. Use Gram-Schmidt process find an orthonormal basis of the subspace of R^4 with standard inner product ,generated by the linearly independent set {(1,1,0,1),(1,1,0,0),(0,1,0,1)}.
18. Test the irreducibility of

    $2x^{10} – 25x^7 +10x^4 +5x^2 +20$ over Q

19. Let (R, +, .) be a ring and f(x), g(x) ∈ $R(x)$.Show that deg (f(x).g(x))≤ deg f(x) +deg g(x).

20. Define prime element with example.

21. Show that x^2 +1 is an irreducible polynomial in Z_3 [x].Hence deduce that the quotient ring $\frac{Z_3[x]}{<x^2+1>}$ is a field of qelement.

22. Let F be a field and an ideal generated by < P(x) >≠<0> In F[x] then <P(x)> is maximal ideal if and only if P(x) is Irreducible over F.

23. Find the minimal solution of x+2y+z =4 ; x-y+2z = -11; x+5y=19.

24. Show that 2 is an irreducible element in z[$\sqrt{-5}$].

25. Test the irreducibility of x^3 + x + 1 in z_5 [x].